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Important Long Questions Mathematics from Past Papers for Inter Part-I

Chapter#02 (Sets, Functions and Groups)

i. Convert $(A \cap B) \cap C = A \cap (B \cap C)$ into logical form and prove by constructing truth table where A, B, Cthree non-empty sets are.
ii. Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$
iii. Prove that all 2×2 non-singular matrices over the real field form a non-abelian group under multiplication.

Chapter#03 (Matrices and Determinants)

i. Prove that $\begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & a+c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$
ii. If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of a and b.
iii. Solve the system of linear equation by Cramer's rule: $2x_1 - x_2 + x_3 = 8;\ x_1 + 2x_2 + 2x_3 = 6;\ x_1 - 2x_2 - x_3 = 1$
iv. Solve the system of linear equation by Cramer's rule: $2x_1 - x_2 + x_3 = 5;\ 4x_1 + 2x_2 + 3x_3 = 8;\ 3x_1 - 4x_2 - x_3 = 3$.
v. Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$
vi. Show that $\begin{vmatrix} b+a & a & a^2 \\ c+b & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$
vii. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ 1 & 3 & 2 \end{bmatrix}$, show that $A + A^t$ is Symmetric.
viii. Show that $\begin{bmatrix} r\cos\emptyset & 0 & -\sin\emptyset \\ 0 & r & 0 \\ r\sin\emptyset & 0 & \cos\emptyset \end{bmatrix} \begin{bmatrix} \cos\emptyset & 0 & -\sin\emptyset \\ 0 & 1 & 0 \\ -r\sin\emptyset & 0 & r\cos\emptyset \end{bmatrix} = rI_3$
ix. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ verify that $(A^{-1})^t = (A^t)^{-1}$

Chapter#04 (Quadratic Equations)

i. Find the values of a and b if -2 and 2 are the roots of polynomial equation $x^3 - 4x^2 + ax + b = 0$.
ii. Solve the system of equations $x^2 - 5xy + 6y^2 = 0;\ x^2 + y^2 = 45$
iii. Solve the equation $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$
iv. If α, β are the roots of $x^2 - 3x + 5 = 0$, form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$
v. Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2, a \neq 0,\ b \neq 0$
vi. Use synthetic division to find the values of p and q if $x+1$ and $x-2$ are the factors of the polynomial $x^3 + px^2 + qx + 6$.
vii. Show that the roots of $(mx+c)^2 = 4ax$ will be equal, if $c = \frac{a}{m};\ m \neq 0$

Chapter#05 (Partial Fractions)

i. Resolve into partial fraction $\frac{x^4}{1-x^4}$
ii. Resolve into partial fractions: $\frac{2x^4}{(x-3)(x+2)^2}$
iii. Resolve into partial fractions: $\frac{1}{(x-1)(2x-1)(3x-1)}$
iv. Resolve into partial fractions: $\frac{9}{(x+2)^2(x-1)}$
v. Resolve $\frac{(x-1)}{(x-2)(x+1)^2}$

Chapter#06 (Sequences and Series)

i. Find four numbers in A.P whose sum is 32 and sum of whose squares is 276.
ii. If $1 + 2x + 4x^2 + 8x^3 + \cdots$ show that $x = \left(\frac{y-1}{2y}\right)$.
iii. The sum of three numbers in A.P is 24 and their product is 440. Find the numbers.
iv. Obtain the sum of all integers in the 1st 1000 integers which are neither divisible by 5 nor by 2.
v. If the H.M and A.M between two numbers are 4 and $\frac{9}{2}$ respectively, find the numbers.
vi. For what value of n, $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive geometric mean between a and b.
vii. Find n, so that $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ may be the A.M. between a and b.
viii. The ratio of the sums of n terms of two series in A.P. is $3n + 2 : n + 1$. Find the ratio of their 8th terms.
ix. If three numbers 1,4 and 3 are subtracted from three consecutive terms of an A.P., the resulting numbers are in G.P. find the number if their sum is 21.
x. The A.M. of two positive integral numbers exceeds their (positive) G.M. by 2 and their sum is 20, find the numbers.
xi. The sum of an infinite geometric series is 9 and the sum of the squares of its terms is $\frac{81}{5}$. Find the series.
xii. Find n, so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the H.M. between a and b.
xiii. If the numbers $\frac{1}{2}, \frac{4}{21}$ and $\frac{1}{36}$ are subtracted from the three consecutive terms of a G.P., the resulting numbers are in H.P. find the numbers if their product is $\frac{1}{27}$.

Chapter#07 (Permutation, Combination, and Probability)

i. A card is drawn from a deck of 52 playing card, what is the probability that it is a diamond card or an ace.
ii. Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
iii. Find the value of n and r when ${}^{n-1}C_{r-1} : {}^{n}C_r : {}^{n+1}C_{r+1} = 3:6:11$
iv. Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$
v. In how many ways can 8 books including 2 on English be arranged on a shelf in such a way that the English books are never together?
vi. How many 6-digit numbers can be formed from the digits 2,2,3,3,4,4? How many of them will lie between 400,000 and 430,000?
vii. ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

Chapter#09 (Fundamentals of Trigonometry)

i. If $cosec\theta = \frac{m^2+1}{2m}$ and $m > 0 \left(0 < \theta < \frac{\pi}{2}\right)$, find the value of remaining trigonometric ratios.
ii. Prove the identity: $sin^6\theta + cos^6\theta = 1 - 3sin^2\theta cos^2\theta$
iii. Find the values of all trigonometric functions of $\frac{19\pi}{3}$.
iv. If $cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in I-quadrant, find the value of $\frac{3sin\theta+4cos\theta}{cos\theta-sin\theta}$
v. Show that $sin^6\theta - cos^6\theta = (sin^2\theta - cos^2\theta)(1 - sin^2\theta cos^2\theta)$
vi. Prove that $\frac{cos\theta+sin\theta}{cos\theta-sin\theta} + \frac{cos\theta-sin\theta}{cos\theta+sin\theta} = \frac{2}{1-2sin^2\theta}$

Chapter#10 (Trigonometric Identities Sum and Difference of Angles)

i. If α, β, γ are the angles of triangle ABC, prove that $tan\frac{\alpha}{2}tan\frac{\beta}{2} + tan\frac{\beta}{2}tan\frac{\gamma}{2} + tan\frac{\gamma}{2}tan\frac{\alpha}{2} = 1$
ii. Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$, given that $tan\alpha = -\frac{15}{8}$ and $sin\beta = -\frac{7}{25}$ and neither the terminal side of the angle of measure α nor that of β is in the IV quadrant.
iii. Prove that $\frac{sin\theta+sin3\theta+sin5\theta+sin7\theta}{cos\theta+cos3\theta+cos5\theta+cos7\theta} = tan4\theta$
iv. Reduce $sin^4\theta$ to an expression involving only function of multiples of θ raised to the first power.
v. Prove without using calculator $sin19^o cos11^o + sin71^o sin11^o = \frac{1}{2}$
vi. Prove that $\frac{2sin\theta sin2\theta}{cos\theta+cos3\theta} = tan2\theta tan\theta$

vii. If $sin\alpha = \frac{4}{5}$ and $sin\beta = \frac{12}{13}$ where $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$. Show that $\sin(\alpha - \beta) = \frac{133}{205}$.

viii. Prove that $cos20^o cos40^o cos60^o cos80^o = \frac{1}{16}$

ix. If α, β, γ are the angles of triangle ABC, prove that $cot\frac{\alpha}{2} + cot\frac{\beta}{2} + cot\frac{\gamma}{2} = cot\frac{\alpha}{2}cot\frac{\beta}{2}cot\frac{\gamma}{2}$

x. If $\alpha + \beta + \gamma = 180^o$, show that $cot\alpha cot\beta + cot\beta cot\gamma + cot\gamma cot\alpha = 1$

xi. $sin\frac{\pi}{9}sin\frac{2\pi}{9}sin\frac{\pi}{3}sin\frac{4\pi}{9} = \frac{3}{16}$

Chapter#12 (Application of Trigonometry)

i. Prove that in an equilateral triangle $r: R: r_1 = 1: 2: 3$

ii. Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

iii. Prove that $r_3 = stan\frac{\gamma}{2}$

iv. Prove that $r = \frac{\Delta}{s}$ with usual notation.

v. Show that $r_3 = 4Rcos\frac{\alpha}{2}cos\frac{\beta}{2}sin\frac{\gamma}{2}$

vi. Show that $\frac{1}{r^2} + \frac{1}{{r_1}^2} + \frac{1}{{r_2}^2} + \frac{1}{{r_3}^2} = \frac{a^2+b^2+c^2}{\Delta^2}$

vii. The sides of a triangle are $x^2 + x + 1$, $2x + 1$, and $x^2 - 1$. Prove that the greatest angle of the triangle is 120^o.

viii. One side of a triangular garden is $30m$. if its two corner angles are $22^o\, {}^1\!/_2$ and $112^o\, {}^1\!/_2$, find the cost of planting the grass at the rate of $Rs.5$ per square meter.

ix. Prove that $(r_1 + r_2)tan\frac{\gamma}{2} = c$

Chapter#13 (Inverse Trigonometric Functions)

i. Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

ii. Prove that $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

iii. Prove that $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$

iv. Prove that $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$

v. Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$

vi. Without using table/calculator show that: $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$