

## Important Long Questions Mathematics from Past Papers for Inter Part-II

### Chapter#01 (Functions and Limits)

1. Evaluate  $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$
2. Evaluate  $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$
3. Evaluate  $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
4. Discuss the continuity of  $f(x)$  at  $x = c$ :  $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}, c = 1$
5. Find the values of  $m$  and  $n$ , so the given function  $f$  is continuous at  $x = 3$ .  $f(x) = \begin{cases} mx - 1 & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$
6. If  $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$  find value of  $k$  so that  $f$  is continuous at  $x = 2$

### Chapter#02 (Differentiation)

1. Differentiate w.r.t 'x',  $\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$
2. Differentiate w.r.t 'x',  $\frac{x\sqrt{a+x}}{\sqrt{a-x}}$
3. If  $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ , show that  $2x \frac{dy}{dx} + y = 2\sqrt{x}$
4. Prove that  $y \frac{dy}{dx} + x = 0$  if  $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$
5. If  $x = a \cos^3 \theta$ ,  $y = b \sin^3 \theta$ , show that  $a \frac{dy}{dx} + b \tan \theta = 0$
6. Show that  $\frac{dy}{dx} = \frac{y}{x}$  if  $\frac{y}{x} = \tan^{-1} \frac{x}{y}$
7. If  $x = \sin \theta$ ,  $y = \sin m\theta$ , show that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$
8. If  $y = e^x \sin x$ , show that  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$
9. If  $y = (\cos^{-1} x)^2$ , prove that  $(1 - x^2)y_2 - xy_1 - 2 = 0$
10. Show that  $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$  and evaluate  $\cos 61^\circ$
11. Show that  $2^{x+h} = 2^x \left\{ 1 + (\ln 2)h + \frac{(\ln 2)^2 h^2}{2!} + \frac{(\ln 2)^3 h^3}{3!} + \dots \right\}$
12. Find the maximum and minimum value of the equation occurring in the interval  $[0, 2\pi]$ .  
 $f(x) = \sin x + \cos x$
13. Show that  $y = \frac{\ln x}{x}$  has maximum value at  $x = e$
14. Show that  $y = x^x$  has minimum value at  $x = \frac{1}{e}$

### Ch#03 (Integration)

1. Evaluate the indefinite integral  $\int \sqrt{4 - 5x^2} dx$
2. Evaluate  $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$
3. Evaluate  $\int \frac{dx}{\sqrt{7-6x-x^2}}$
4. Evaluate  $\int e^{2x} \cos 3x dx$
5. Solve  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$
6. Evaluate  $\int \frac{2x}{1-\sin x} dx$

7. Evaluate the indefinite integral  $\int \sqrt{x^2 - a^2} dx$
8. Evaluate  $\int \ln(x + \sqrt{x^2 + 1}) dx$
9. Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$
10. Evaluate  $\int x \sin^{-1} x dx$
11. Evaluate  $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin x(2 + \sin x)} dx$
12. Find the area between the x-axis and curve  $y = \sqrt{2ax - x^2}$ , where  $a > 0$
13. Evaluate  $\int_0^{\pi/4} \cos^4 t dt$
14. Evaluate  $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

### **Chapter#04 (Introduction to Analytic Geometry)**

1. Find the lines represented by  $9x^2 + 24xy + 16y^2 = 0$  and also find measure of the angle between them.
2. Find equation of line through intersection of lines  $x - y - 4 = 0$ ,  $7x + y + 20 = 0$  and parallel to the line  $6x + y - 14 = 0$
3. Find h such that the points  $A(h, 1)$ ,  $B(2, 7)$ ,  $C(-6, -7)$  are the vertices of right triangle with right angle at vertex A.
4. Find equation of two parallel lines perpendicular to  $2x - y + 3 = 0$  such that the product of x-intercept and y-intercept is 3.
5. Find the distance between the given parallel lines. Sketch the lines. Also find an equation of the parallel lines lying midway between them.  $3x - 4y + 3 = 0$ ;  $3x - 4y + 7 = 0$

### **Chapter#05 (Linear Inequalities and Linear Programming)**

1. Graph the feasible region of the linear inequalities and find corner points.  
 $2x + 3y \leq 18$ ;  $x + 3y \leq 10$ ;  $x + 4y \leq 12$
2. Minimize  $z = 3x + y$ ; subject to the constraints  $3x + 5y \geq 15$ ;  $x + 3y \geq 9$ ;  $x, y \geq 0$
3. Maximize  $f(x, y) = x + 3y$  subject to the constraints  $2x + 5y \leq 30$ ;  $5x + 4y \leq 20$ ;  $x \geq 0, y \geq 0$
4. Maximize  $f(x, y) = 2x + 5y$  subject to the constraints  $2y - x \leq 8$ ;  $x - y \leq 4$ ;  $x \geq 0, y \geq 0$
5. Graph the feasible region of the linear inequalities and find corner points.  
 $x + y \leq 5$ ;  $-2x + y \leq 2$ ;  $x \geq 0, y \geq 0$

### **Chapter#07 (Vectors)**

1. Prove that the angle in a semi-circle is a right angle.
2. Use vector method to show that  $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. Find a unit vector perpendicular to the plane containing  $\underline{a}$  and  $\underline{b}$ . also find the sine of angle between them.  
 $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$ ;  $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$
4. Prove that the line segment joining the mid points of two sides of a triangle is parallel to third side and half as long.
5. Prove that the line segment joining the mid points of the sides of a quadrilateral taken in order form a parallelogram.
6. Prove that the perpendicular bisector of the sides of a triangle are concurrent.
7. Prove that the altitudes of a triangle are concurrent.
8. Prove that  $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
9. Prove that in any triangle  $\Delta ABC$ .  
(i)  $b = c\cos A + a\cos C$ ; (ii)  $c = a\cos B + b\cos A$   
(iii)  $b^2 = c^2 + a^2 - 2ac\cos B$ ; (iv)  $c^2 = a^2 + b^2 - 2ab\cos C$